A Constrained Resampling Strategy for Mesh Improvement

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Problem Definition

Improving an input mesh in terms of a given set of quality objectives

How to translate the quality objectives

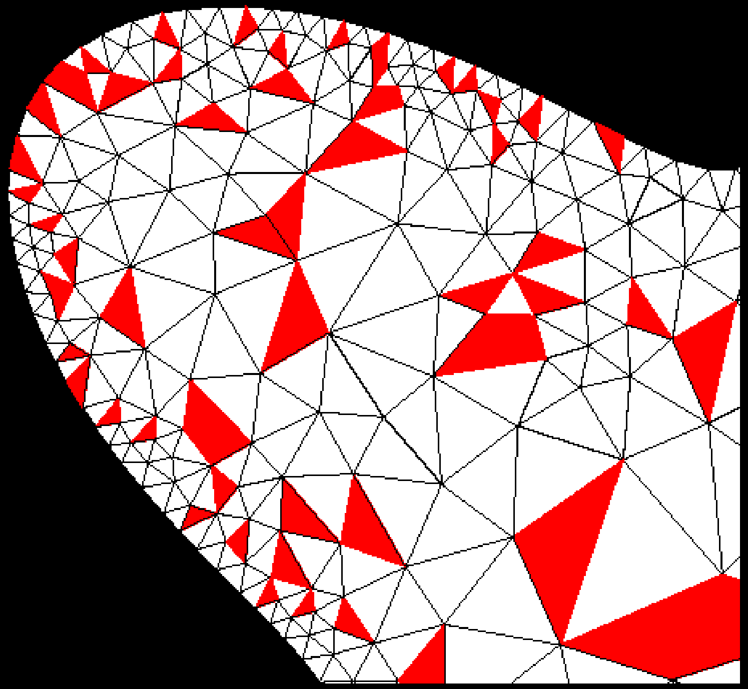
How to achieve multiple objectives

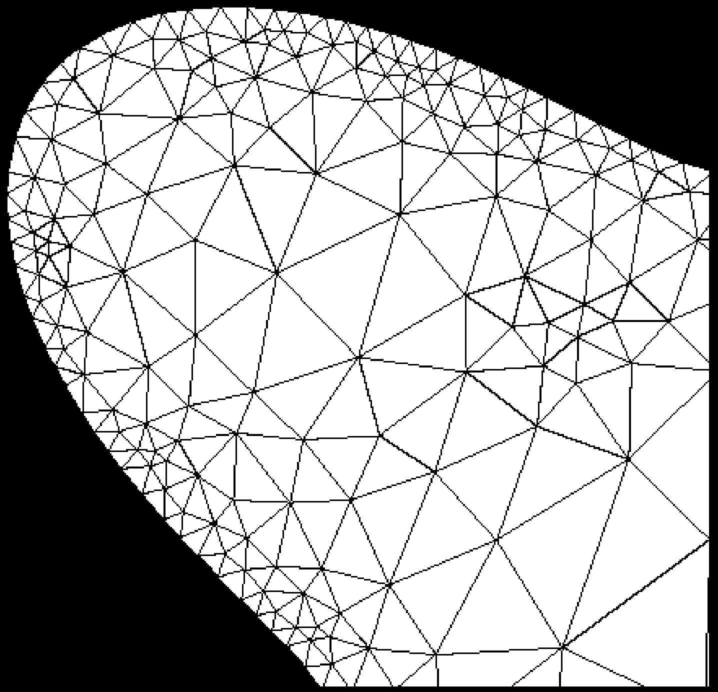
Applications

1- Non-obtuse Triangulation:

Input: obtuse triangular mesh

Target: Eliminate all obtuse triangles

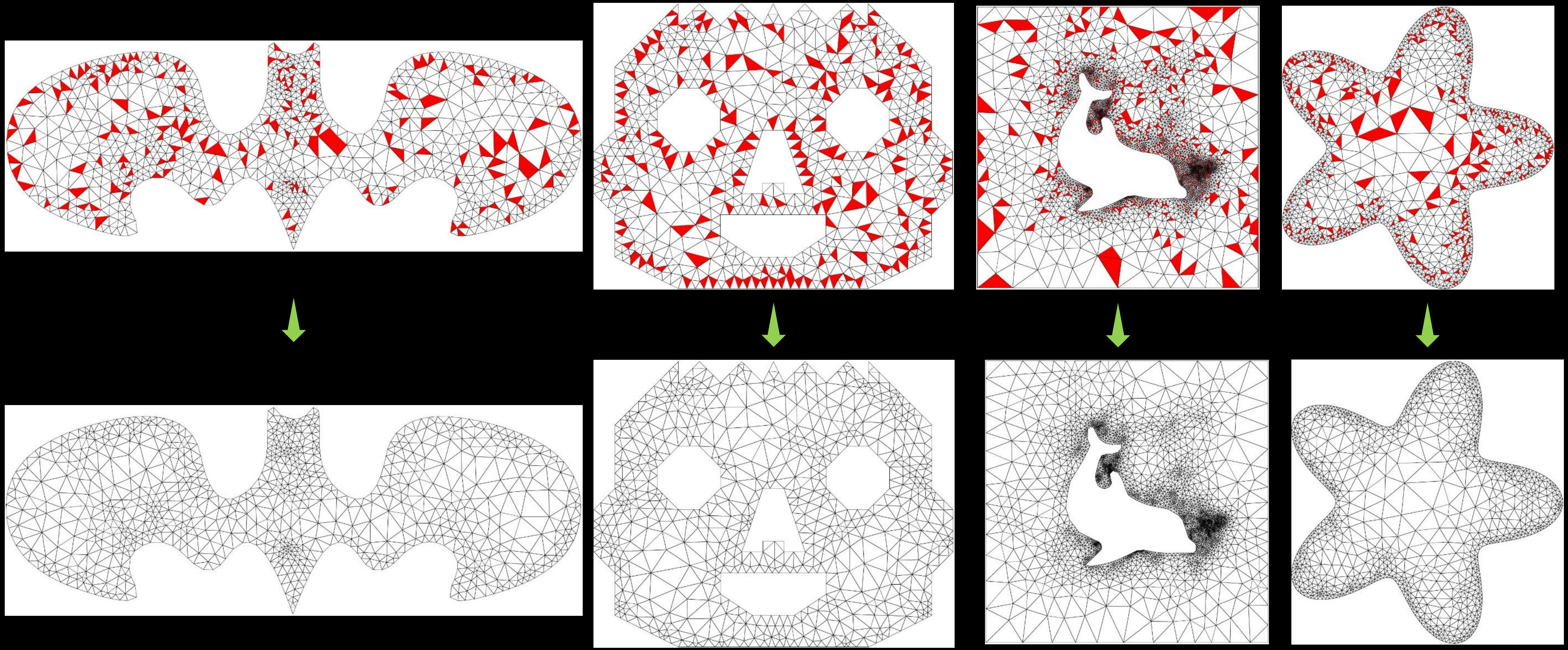




Red = obtuse

Applications

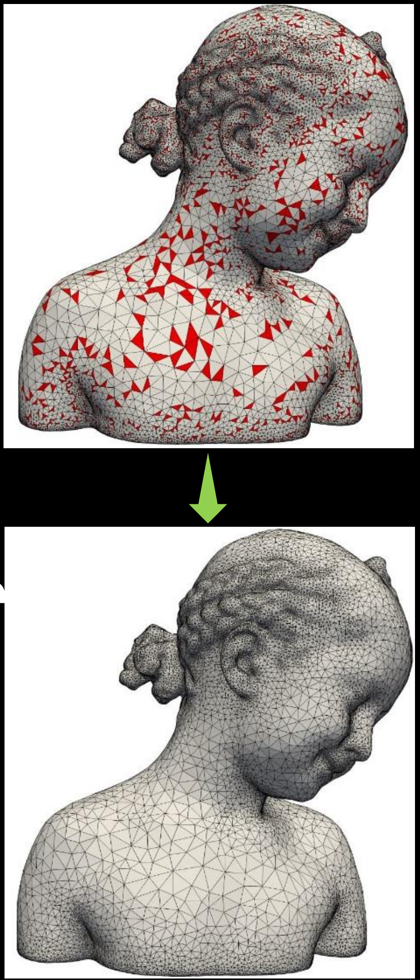
1- Non-obtuse Triangulation (2D)



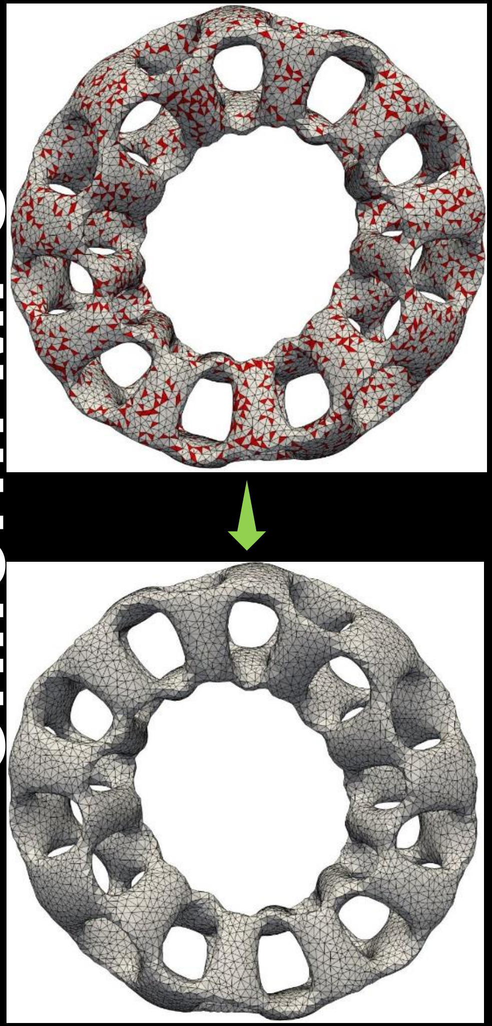
Applications

1- Non-obtuse Triangulation (CS)

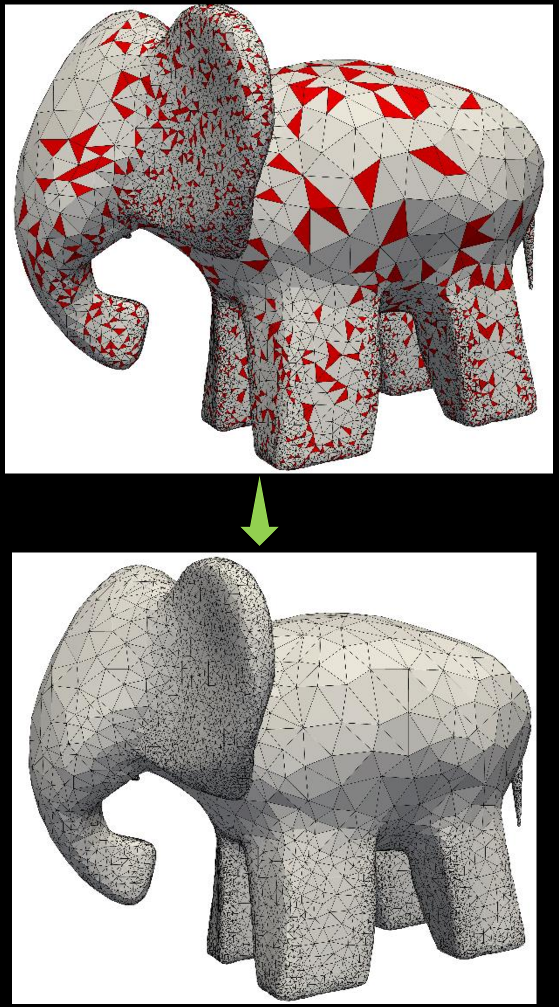
Delaunay Refinement



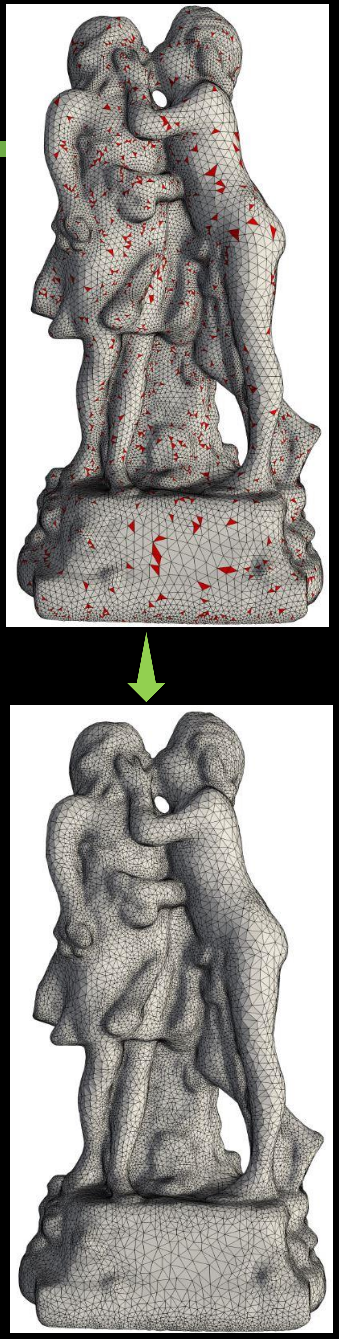
Uniform MPS



Non-uniform MPS



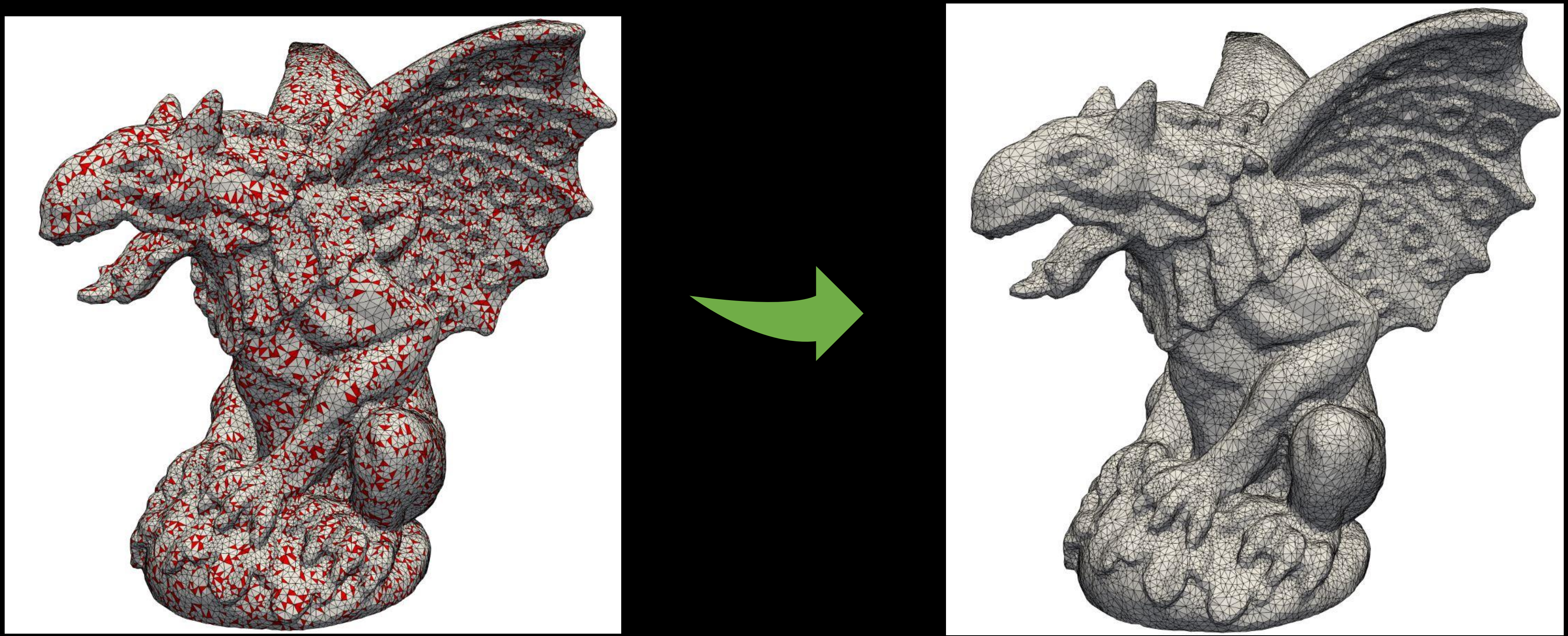
Frontal Delaunay



Applications

1- Non-obtuse Triangulation (CS)

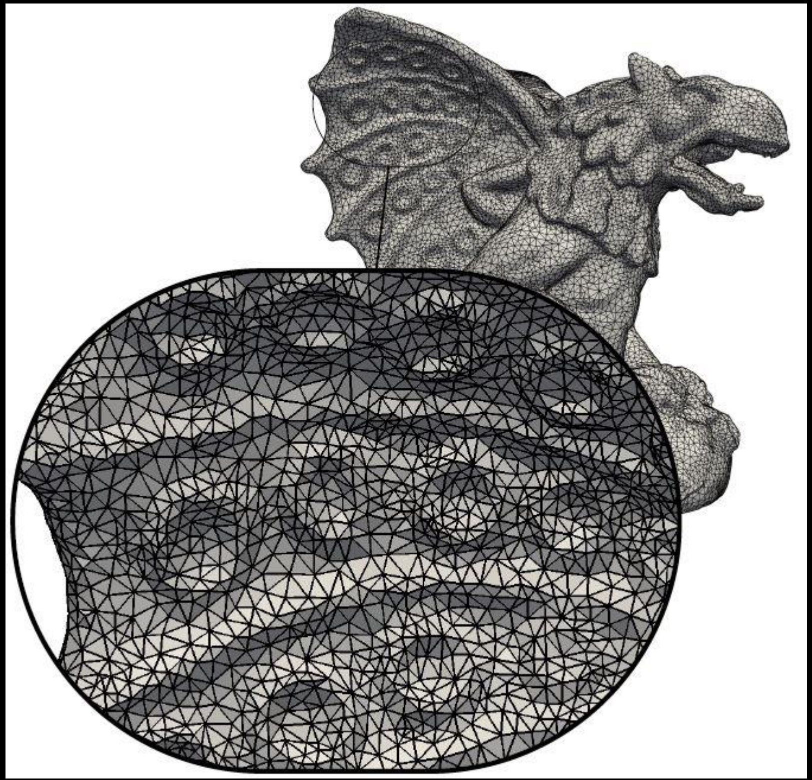
Comparison: “Non-obtuse remeshing with centroidal voronoi tessellation ” – *D.M. Yan et al., IEEE TVCG 2016*

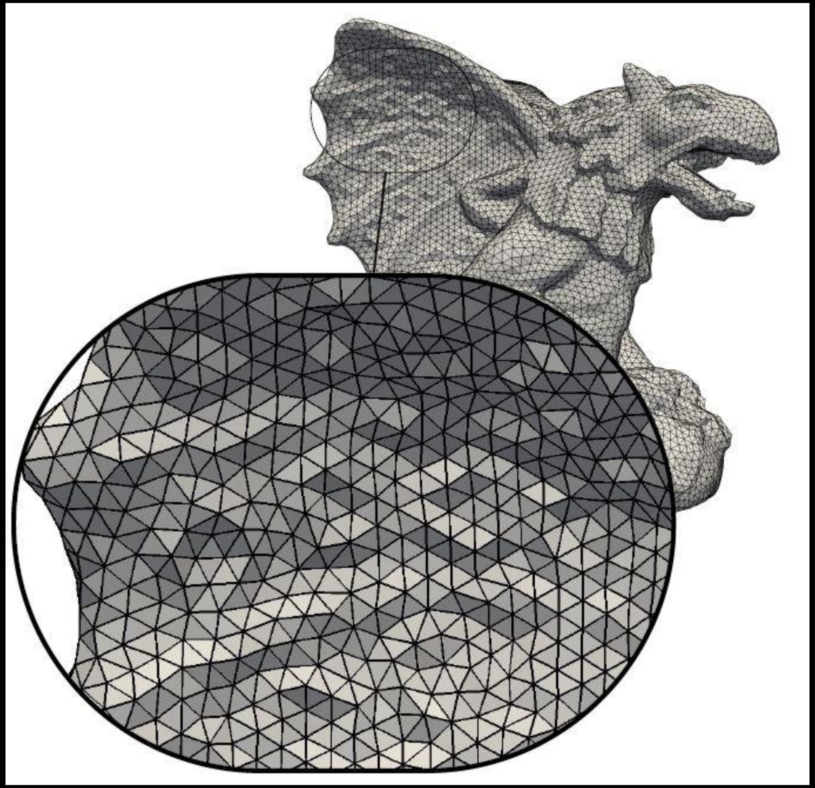


Applications

1- Non-obtuse Triangulation (CS)

Comparison: "A simple pull-push algorithm for blue-noise sampling" - *AG Ahmed et al., IEEE TVCG 2016*



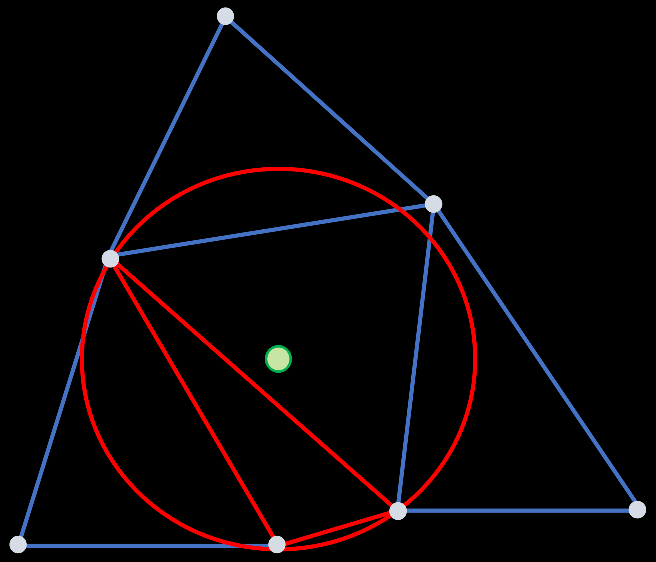


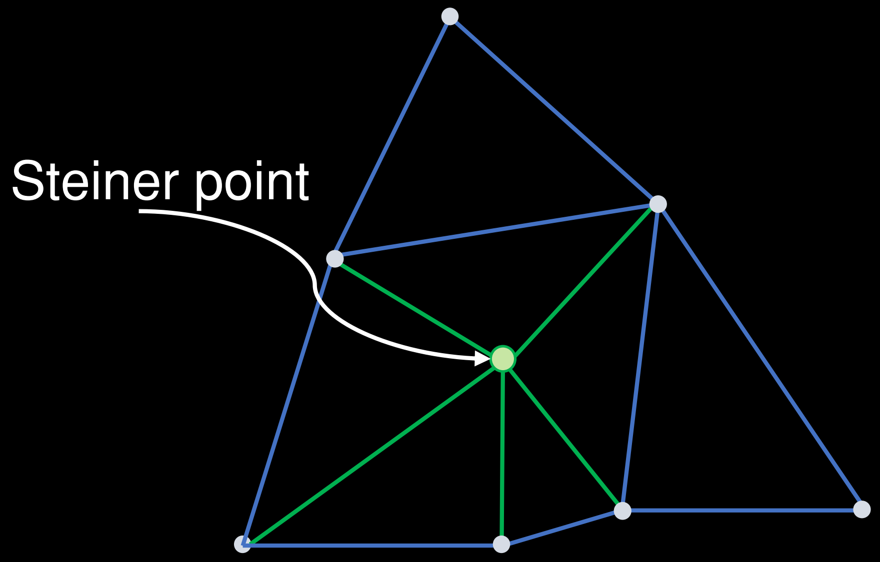
Applications

2- Delaunay Sifting:

Definition: Reducing the number of Steiner points while preserving the same qualities of the input mesh

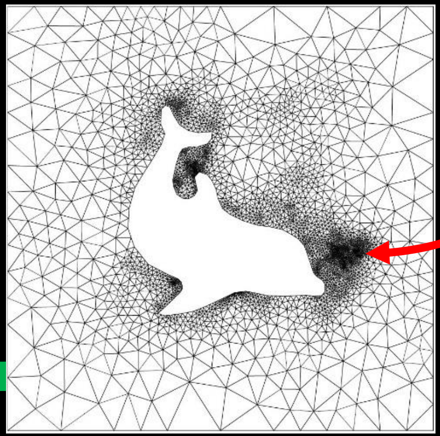
Steiner points: Set of vertices inserted in initial Delaunay mesh to improve its quality (minimum angle, triangle area)



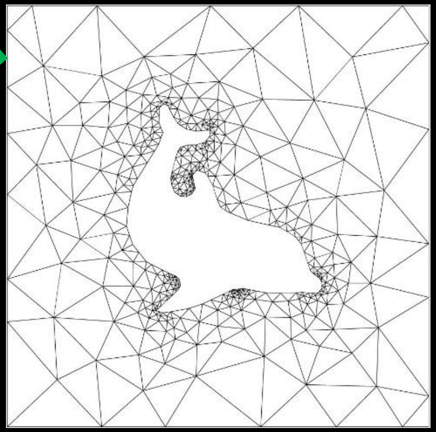


Applications

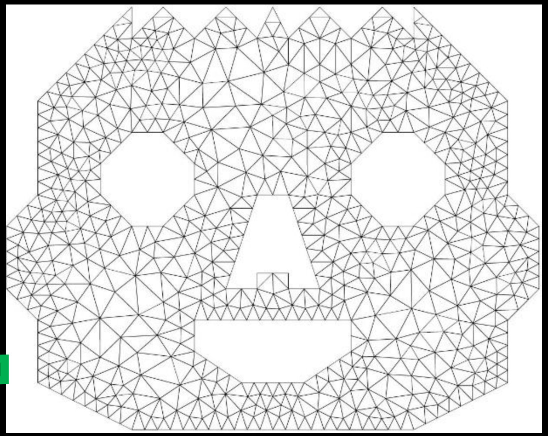
2- Delaunay Sifting (2D) *Triangle*



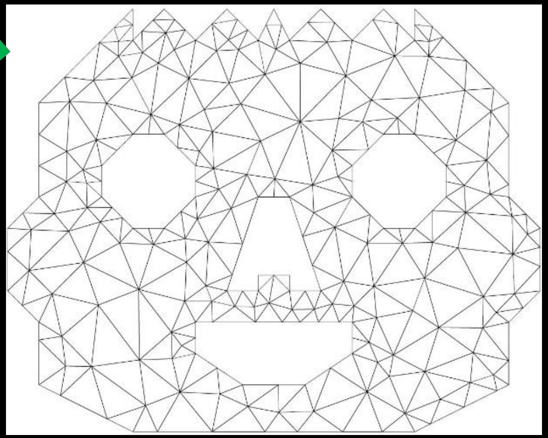
$\theta_{min} = 35°, \theta_{max} = 109°$



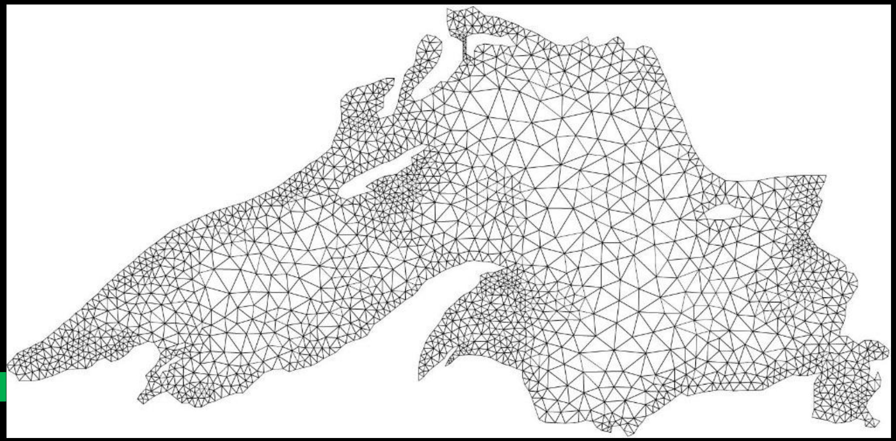
Reduction ratio = 86%



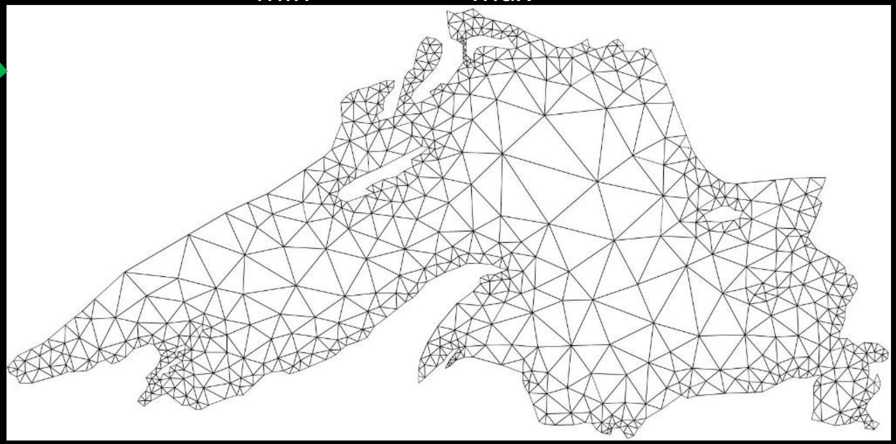
$\theta_{min} = 35°, \theta_{max} = 110°$



Reduction ratio = 62%



$\theta_{min} = 35°, \theta_{max} = 109°$

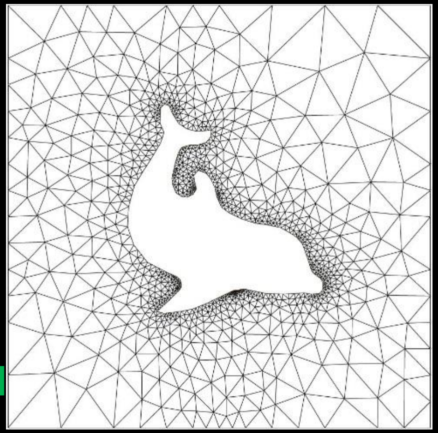


Reduction ratio = 61%

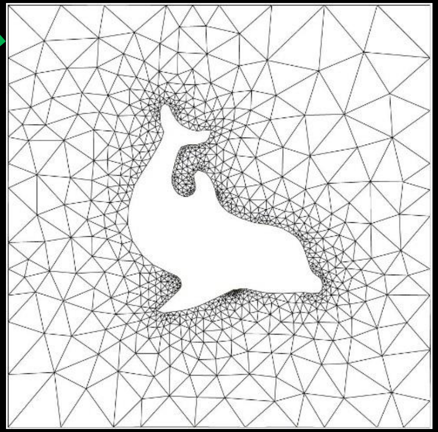
Applications

2- Delaunay Sifting (2D) *aCute*

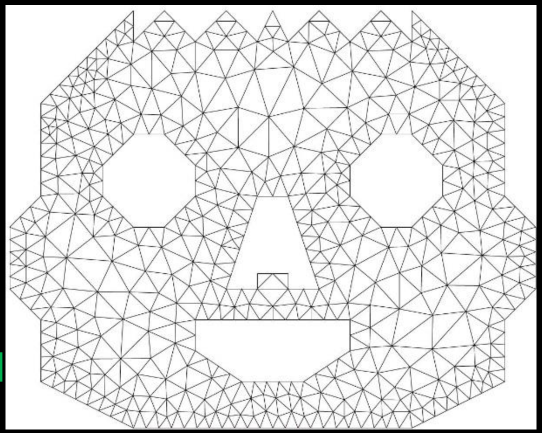
"Quality Triangulations with Locally Optimal Steiner Points" – *Hale Erten et al.* - SIAM J. Sci. Comput 2008



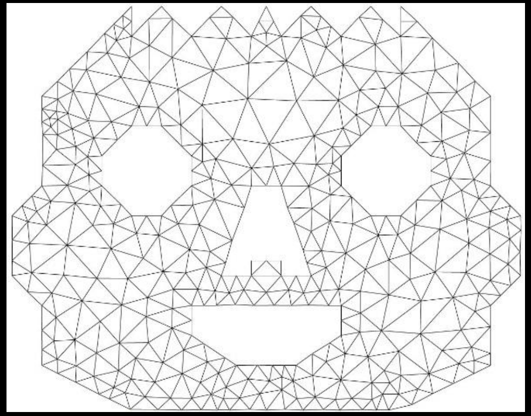
θ_{min} =40°, θ_{max} =99°



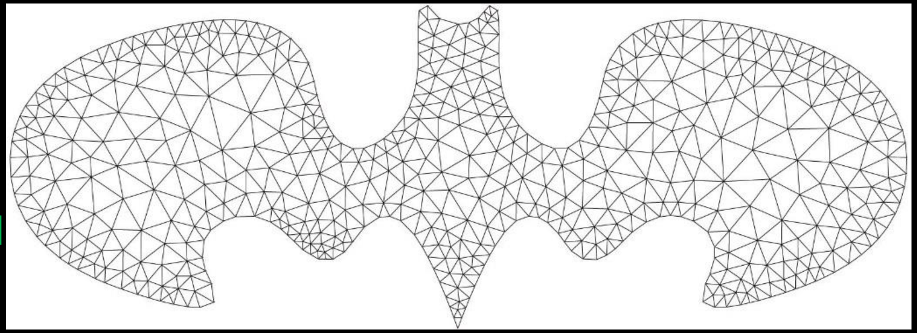
Reduction ratio = 24%



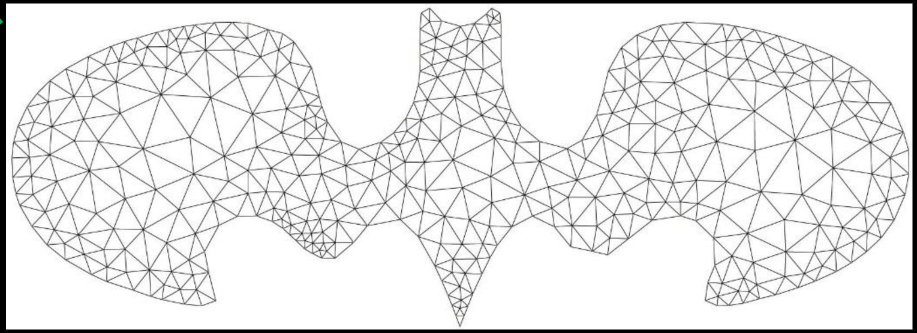
θ_{min} =40°, θ_{max} =99°



Reduction ratio = 28%



θ_{min} =40°, θ_{max} =99°



Reduction ratio = 28%

Applications

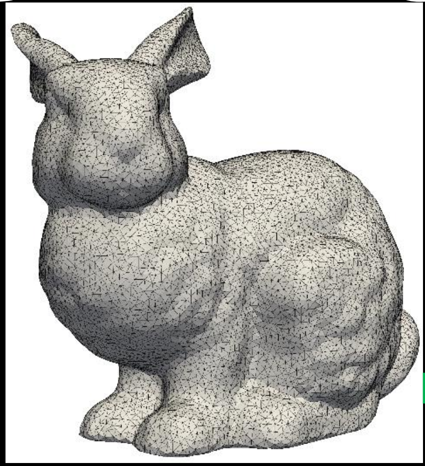
2- Delaunay Sifting

CVT

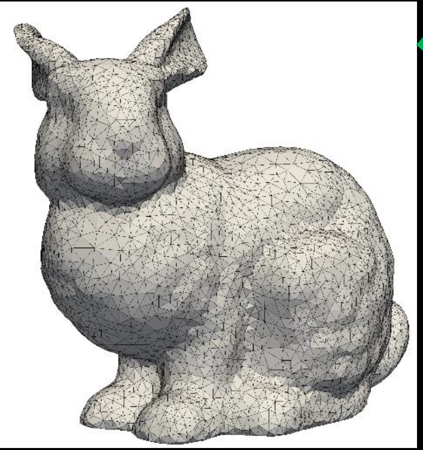
$\theta_{min} = 35°, \theta_{max} = 103°$

Reduction ratio = 43%

Uniform MPS

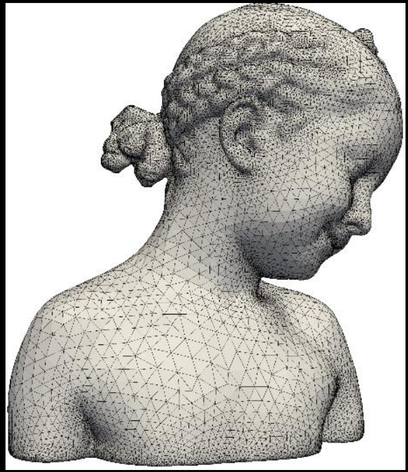


$\theta_{min} = 30°, \theta_{max} = 116°$

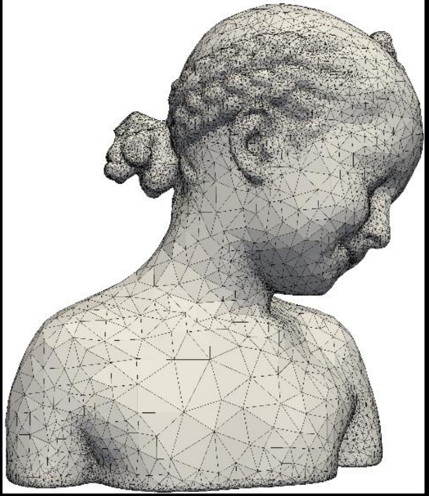


Reduction ratio = 50%

Frontal Delaunay

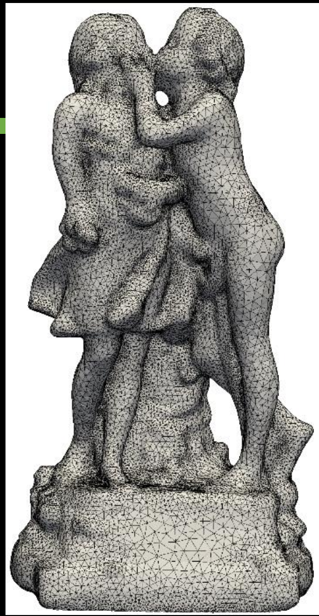


$\theta_{min} = 28°, \theta_{max} = 120°$

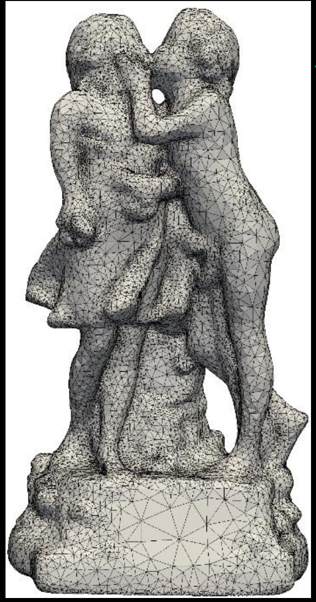


Reduction ratio = 50%

Delaunay Refinement



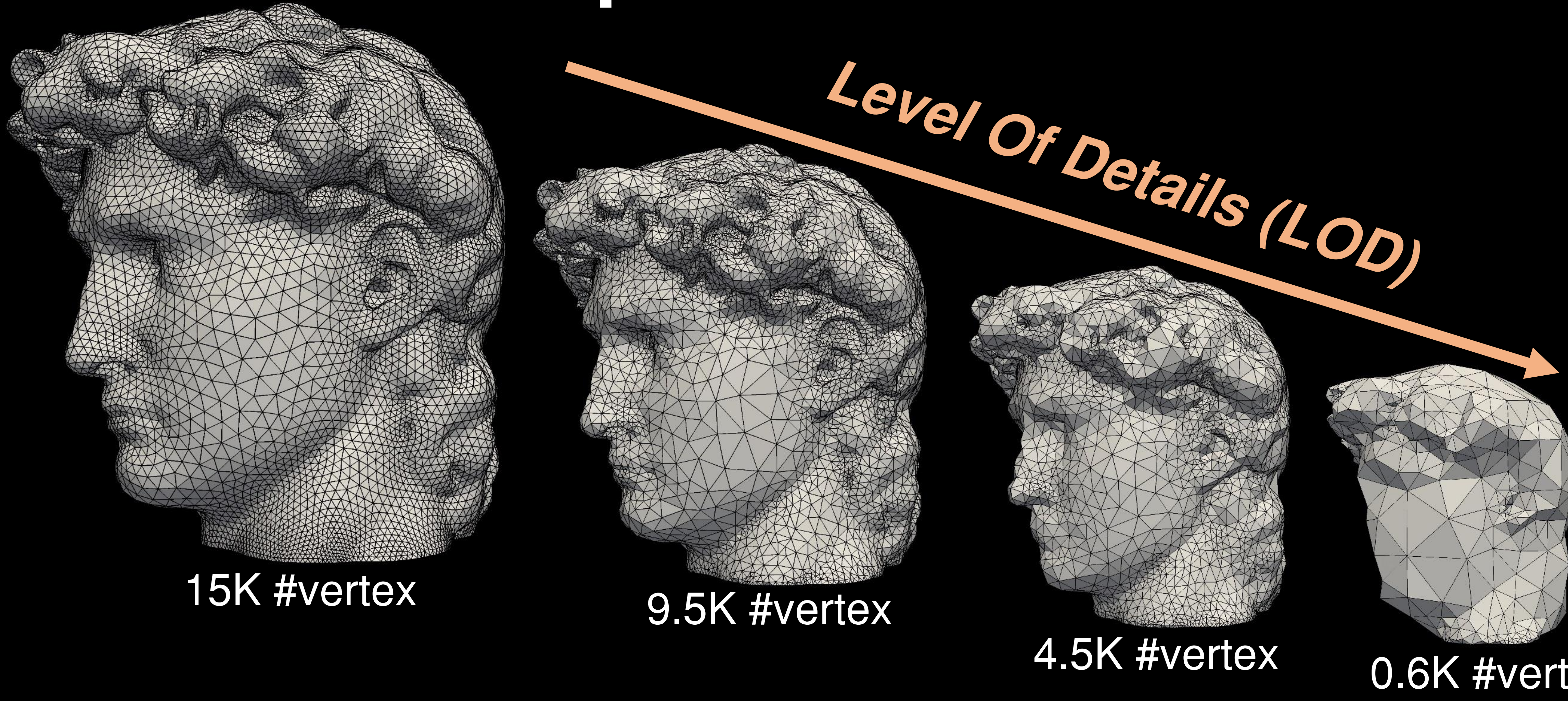
$\theta_{min} = 28°, \theta_{max} = 121°$



Reduction ratio = 44%

Applications

3- Mesh Simplification



Applications

3- Mesh Simplification

- “Surface Simplification Using Quadratic Error Metric” – *Garland M.,et al.* - SIGGRAPH '97
- “Efficient Construction and Simplification of Delaunay Meshes” – *Liu Y.-J., et al.* - TOG 2015

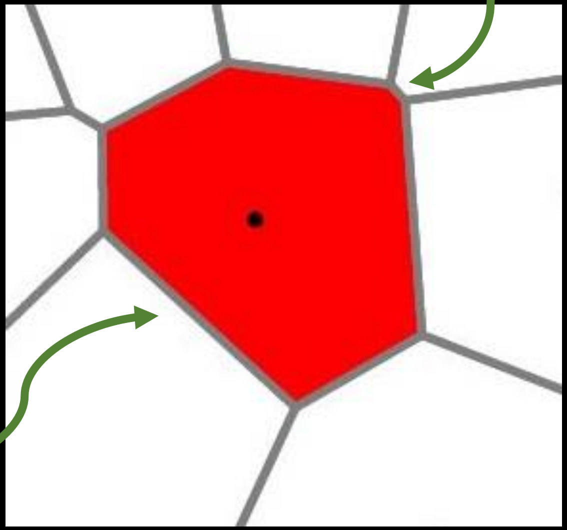
Min Angle
Max Angle
Hausdorff Distance

Model	Method	v		$\triangle$		θ_{min}		θ_{max}		Q_{min}		$d_{RMS}(\times10^{-2})$	$d_H(\times10^{-2})$
		Input	Output	Input	Output	Input	Output	Input	Output	Input	Output		
	DM						12		171		0.06	3.5	0.8
Bunny (MPS)	QEM	11.5K	≈153	23k	≈302	30	6	116	165	0.5	0.11	**1.7**	**0.5**
	Our						**30**		**116**		**0.5**	4.6	1.8
	DM						4.5		155		0.12	1.5	0.4
Fertility (MPS)	QEM	8.5K	≈390	17k	≈790	30	4	116	168	0.5	0.08	**0.7**	**0.2**
	Our						**30**		**116**		**0.5**	4.86	0.9
	DM						10		159		**0.2**	1	0.2
Loop (MPS)	QEM	10.7K	≈1.4K	22k	≈3K	30	5.5	117	160	0.48	0.12	**0.5**	**0.1**
	Our						**30**		**117**		0.17	2.9	0.4
	DM						12		137		0.31	4.6	0.6
Bimba (DR)	QEM	25.4K	≈180	51k	≈350	28	6	122	161	0.44	0.12	**1.9**	**0.4**
	Our						**28**		**122**		**0.47**	4.7	1.5
	DM						13		135		0.31	2.3	0.4
Rocker (DR)	QEM	10.8K	≈240	21k	≈485	30	4	118	165	0.47	0.09	**1.1**	**0.3**
	Our						**30**		**114**		**0.5**	4.9	1.3
	DM						11		132		0.3	3.3	0.4
Bimba (FD)	QEM	24.4K	≈270	49K	≈535	28	5	121	167	0.46	0.08	**0.8**	**0.3**
	Our						**29**		**119**		**0.47**	4.8	1.2
	DM						9		130		0.24	1.9	0.4
Rocker (FD)	QEM	10.2K	≈260	20.5k	≈520	32	5	114	167	0.52	0.09	**0.8**	**0.3**
	Our						**32**		**112**		**0.52**	4.9	1.1
	DM						11		138		0.27	1.5	0.2
Chinese Dragon (CVT)	QEM	30K	≈1.5K	60k	≈3.1K	34	3	103	174	0.6	0.04	**0.7**	**0.1**
	Our						**34**		**103**		**0.6**	2.7	0.4
	DM						8		151		0.2	1.1	**0.2**
David Head (CVT)	QEM	15K	≈660	30k	≈1.3K	33	7	107	158	0.56	0.15	**1.9**	**0.2**
	Our						**33**		**107**		**0.56**	4.9	0.9
	DM						12		140		0.3	2.9	0.5
Omotondo (CVT)	QEM	20K	≈260	40k	≈530	28	7	110	142	0.54	0.18	**1.1**	0.3
	Our						**28**		**110**		**0.53**	1.8	**0.2**

Applications

4- Voronoi without Short Edges:

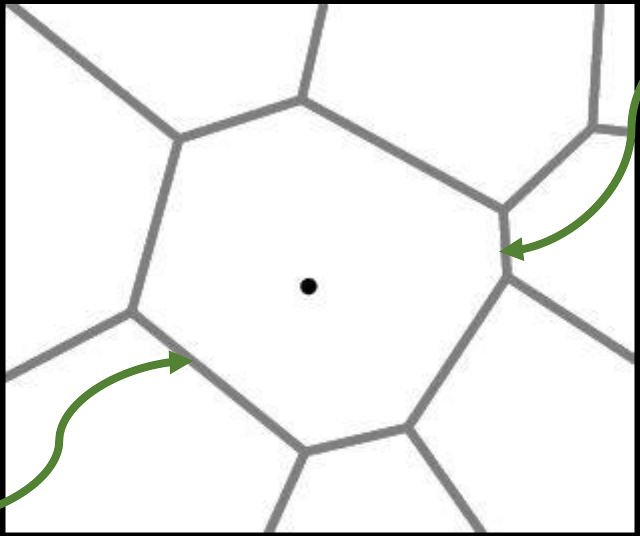
*Shortest Voronoi Edge within the cell |**e1**|*



*Longest Voronoi Edge within the cell |**e2**|*

Bad Voronoi Cell :=
*|**e1**|/ |**e2**| <0.1*

*Shortest Voronoi Edge within the cell |**e1**|*

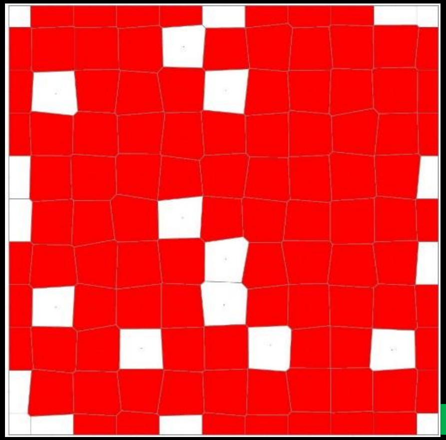


*Longest Voronoi Edge within the cell |**e2**|*

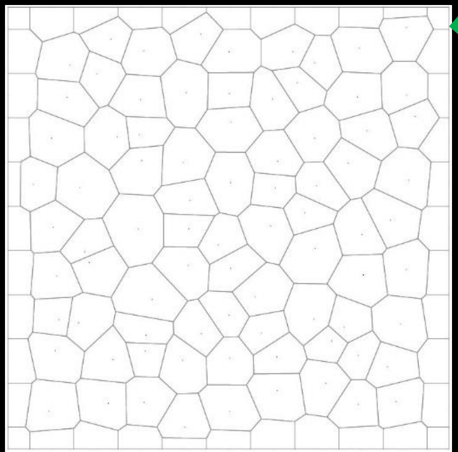
Good Voronoi Cell :=
*|**e1**|/ |**e2**| >0.1*

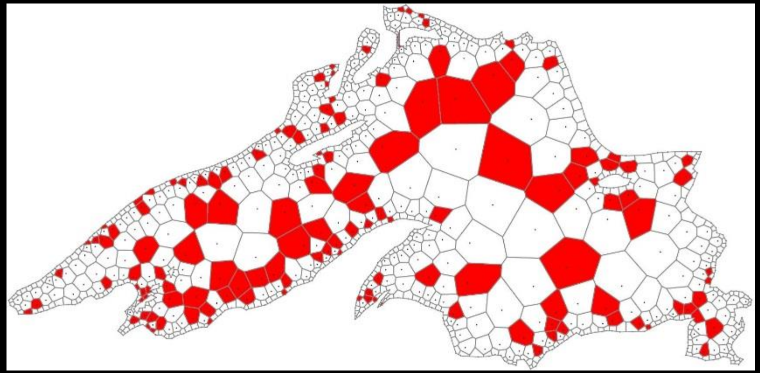
Applications

4- Voronoi without Short Edges:

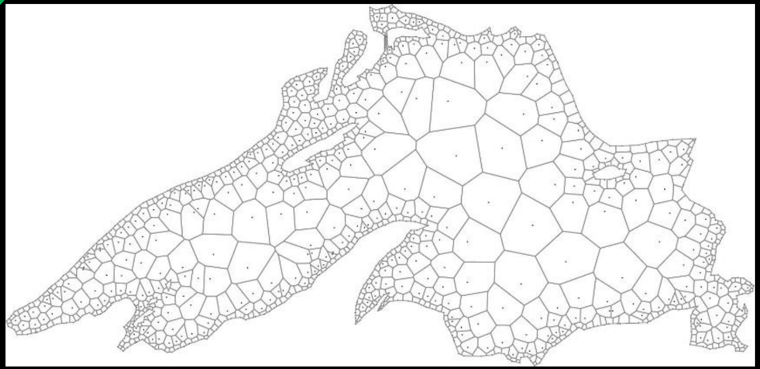


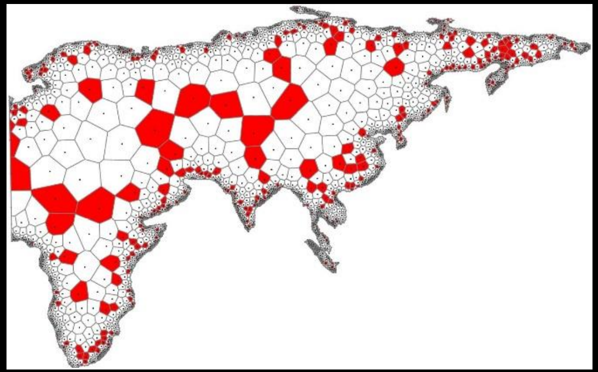
Jittered grid
(98 bad elements)

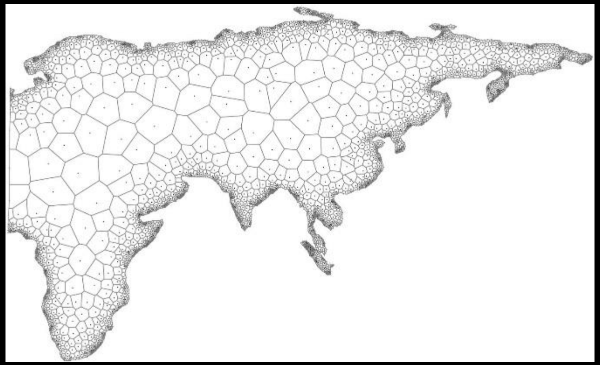




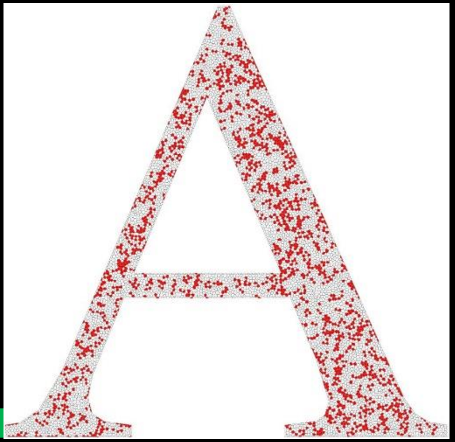
Rapid change in grading
(139 & 541 bad elements)

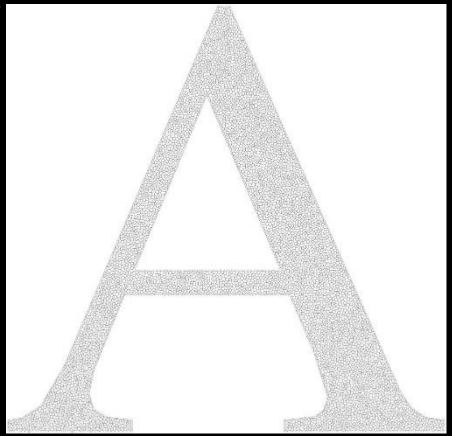






Constant sizing func
(1666 bad elements)





Applications

4- Voronoi without Short Edges:

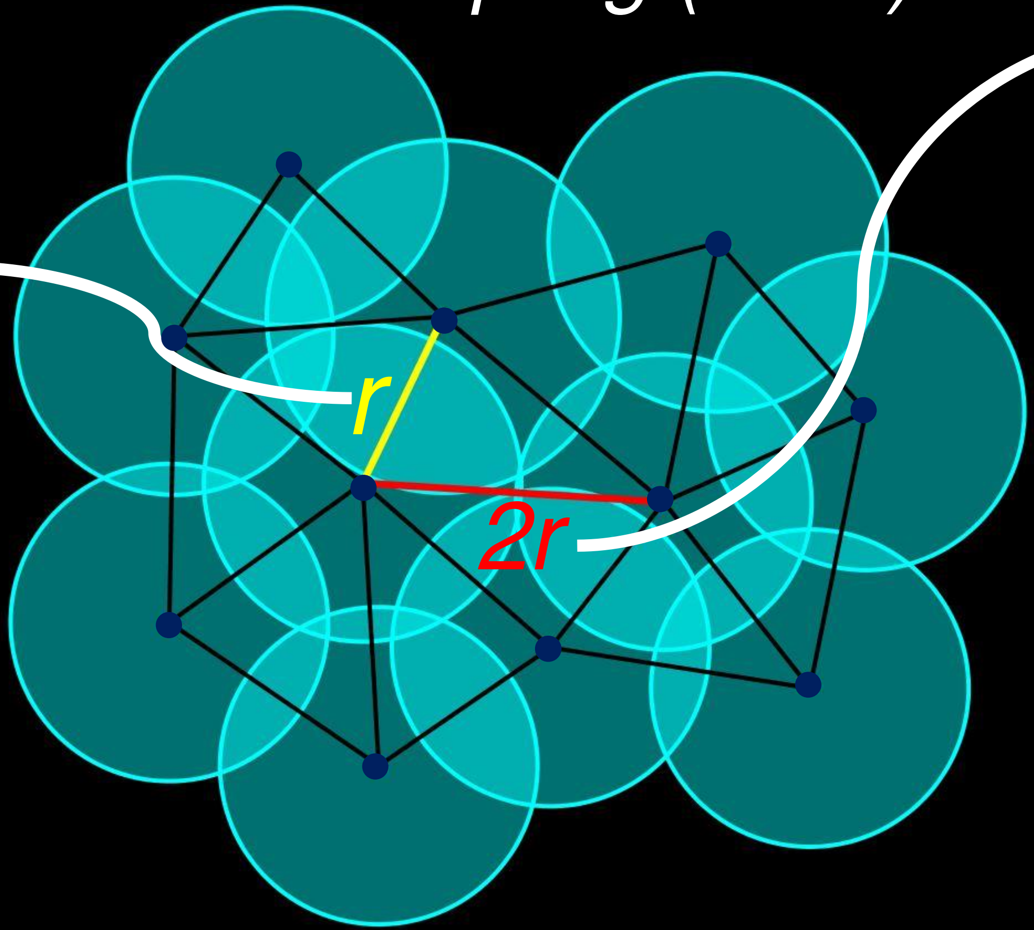
Gray-scale based Voronoi mesh
(14272 bad elements)

The Strategy

Intuition:

- *Maximal Poisson Sampling (MPS)*

Minimum separation ensures minimum edge length



Maximality ensures upper bound on edge length

The Strategy

Input:

$\underbrace{\textit{Triangular mesh}}_{}$ *&* $\underbrace{\textit{quality objectives}}_{}$

Curved Surface mesh
Planar 2D Mesh

Minimum and maximum angle bound
Delaunay property
Sizing function

The Strategy

Definitions:

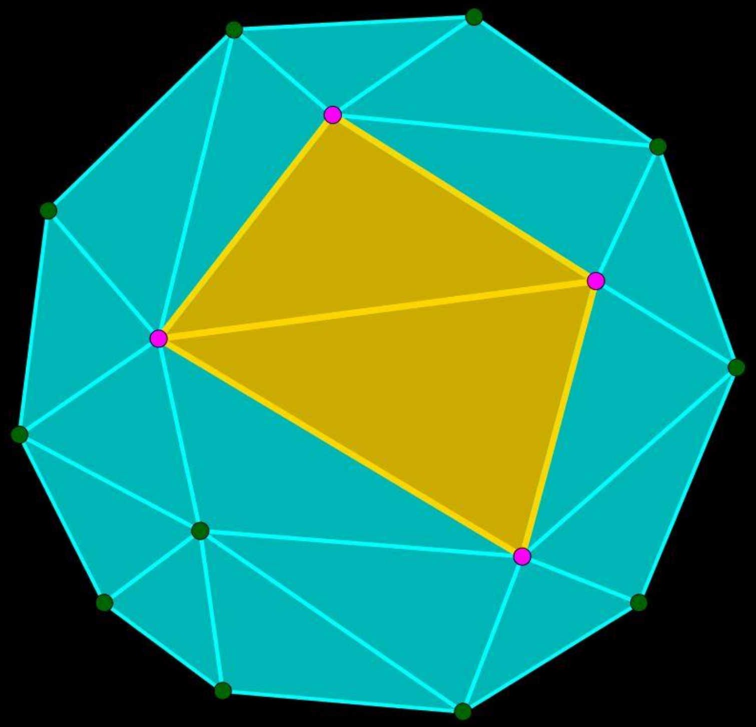
- *Bad element:*

e.g., obtuse triangle for non-obtuse remeshing,
Voronoi cell associated with a short edge,
any triangle for mesh simplification

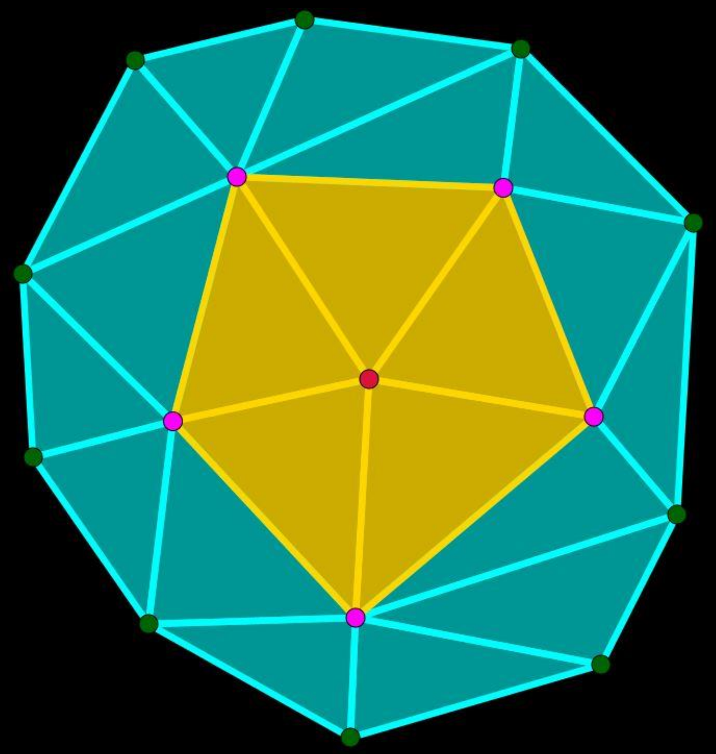
The Strategy

Definitions:

- *Patch:*



Two Opposite Triangles

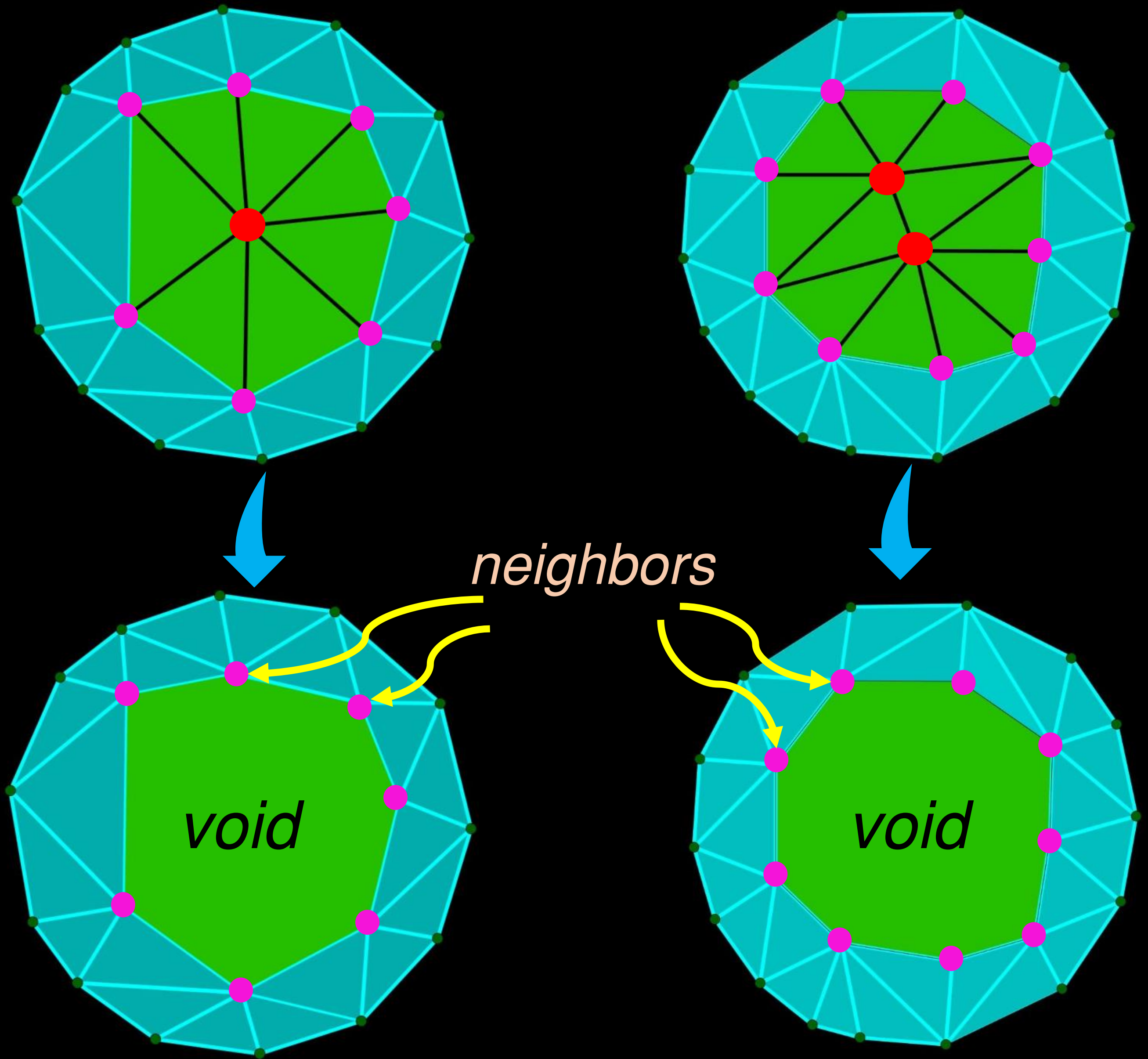


Triangle Fan

The Strategy

Definitions:

- *Void:*



The Strategy

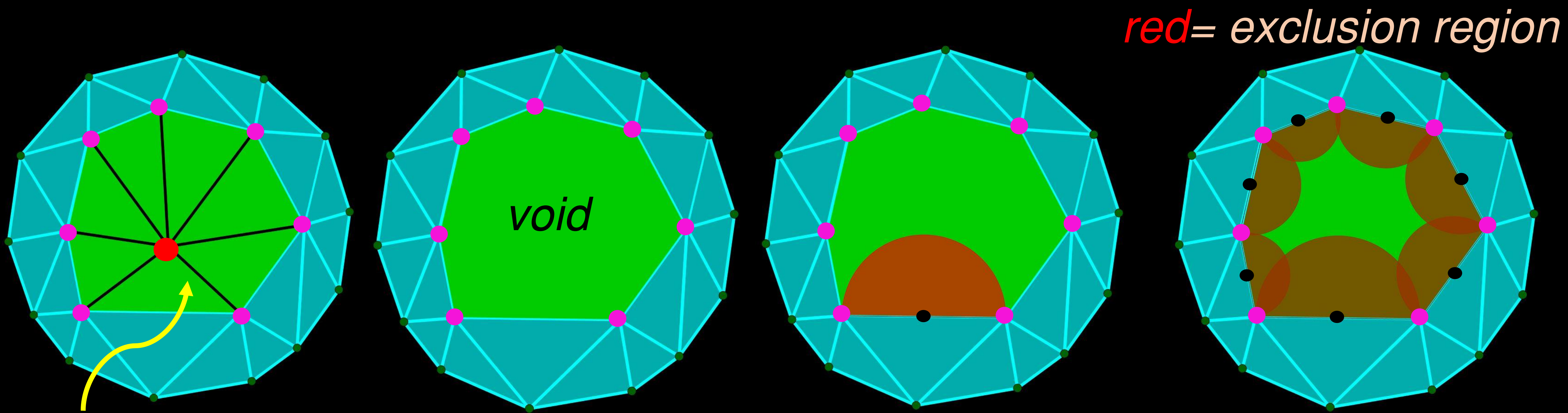
Steps:

1- Pick a patch where quality objectives not satisfied
2- Delete all elements on this patch (*void*)
3- Map quality objective into geometric constraints (feasible region)
4- Sample from the feasible region and triangulate
5- Iterate over all mesh patches until no further improvement is possible

The Strategy

Quality Objectives → Geometric Primitives:

a) Exclusion region:



patch associated with bad element

removing patch elements creates void

map quality objective (non-obtuseness) for a single segment

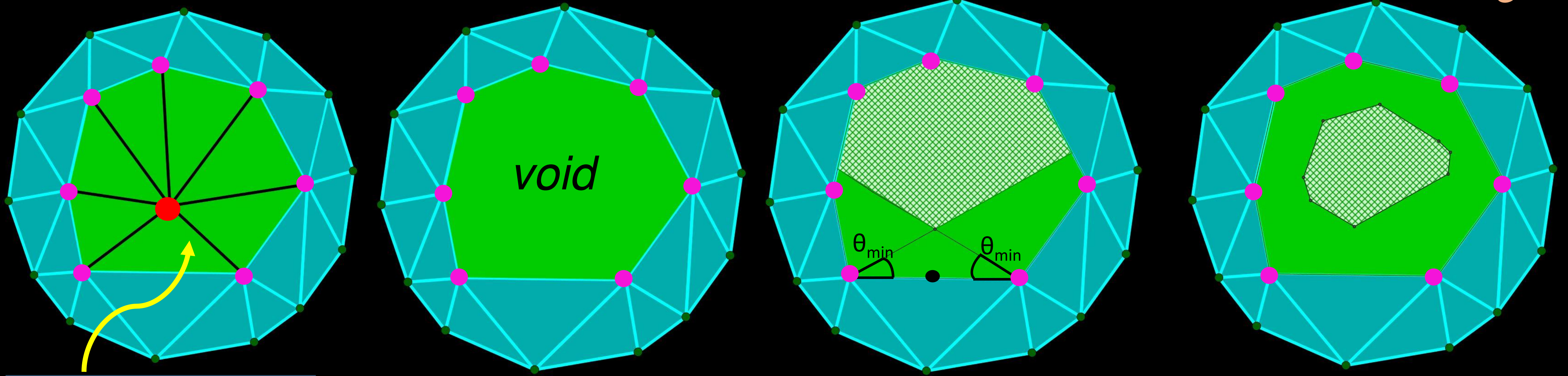
map quality objective from all segments

The Strategy

Quality Objectives → Geometric Primitives:

a) Inclusion region:

hatch= inclusion region



patch associated with bad element

removing patch elements creates void

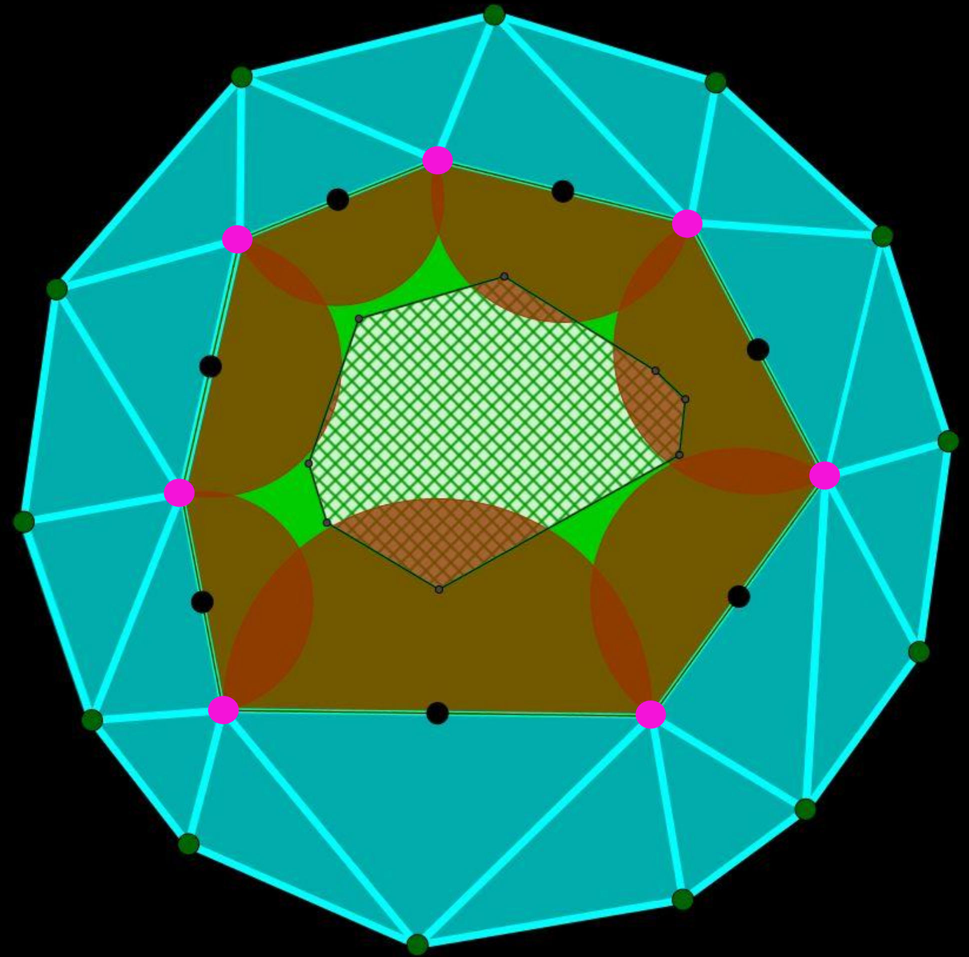
map quality objective (min angle) for a single segment

map quality objective from all segments

The Strategy

Quality Objectives → Geometric Primitives:

Exclusion & Inclusion regions:



hatch= inclusion region

red = exclusion region

The Strategy

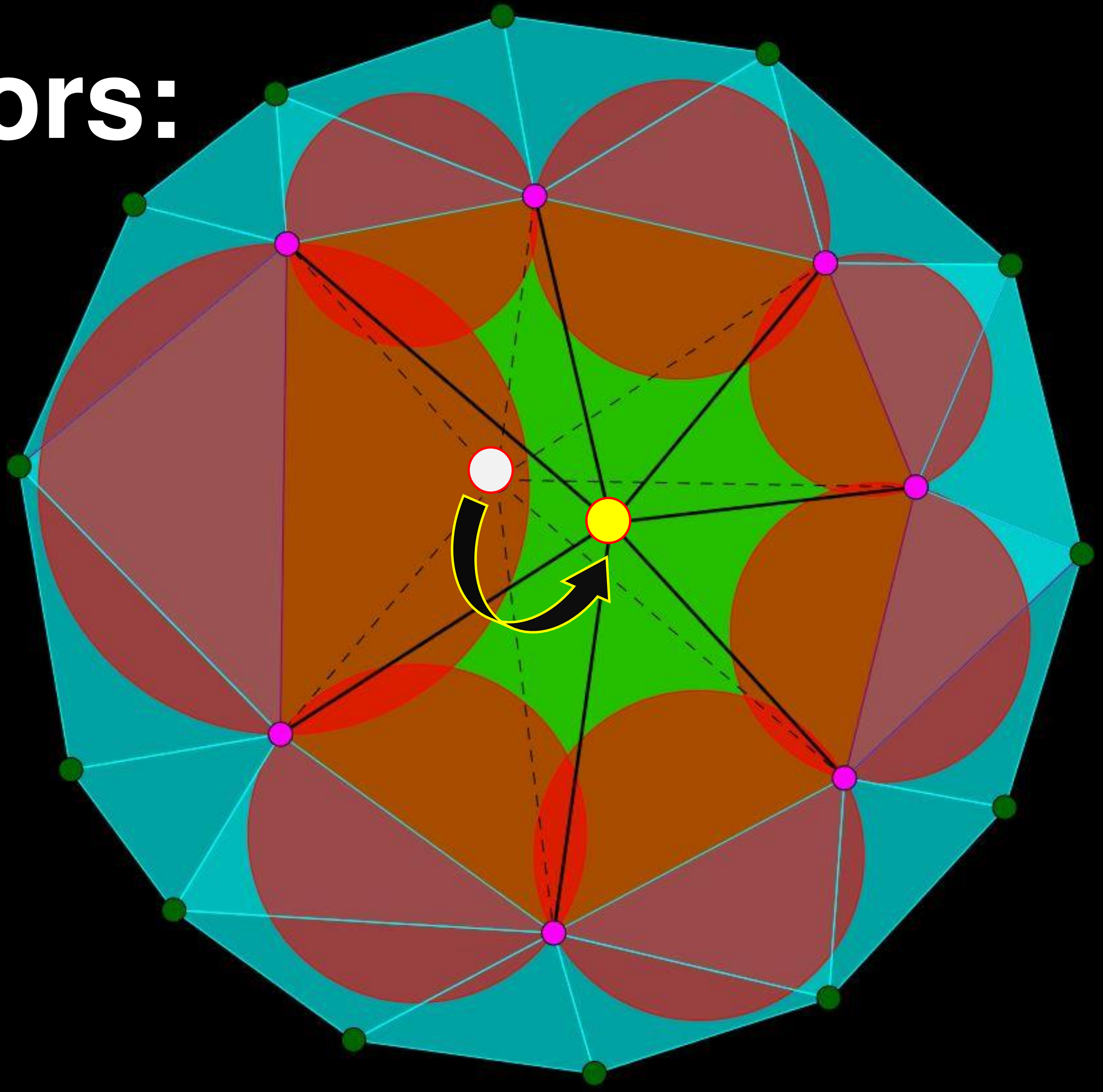
Steps:

1- Pick a patch where quality objectives not satisfied
2- Delete all elements on this patch (*void*)
3- Map quality objective into geometric constraints (feasible region)
4- Sample from the feasible region and triangulate
5- Iterate over all mesh patches until no further improvement is possible

The Strategy

Resampling Operators:

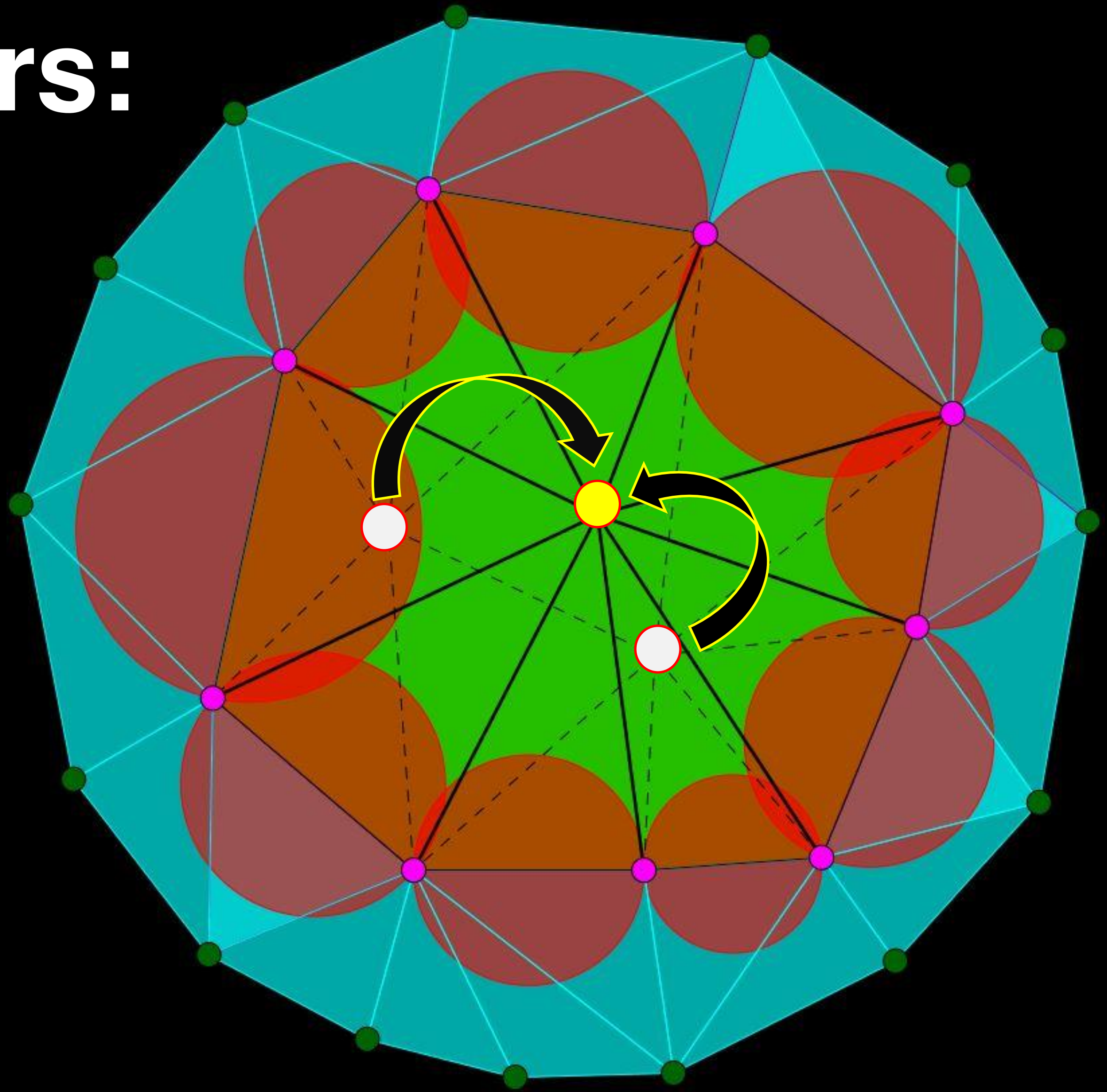
1) Relocation



The Strategy

Resampling Operators:

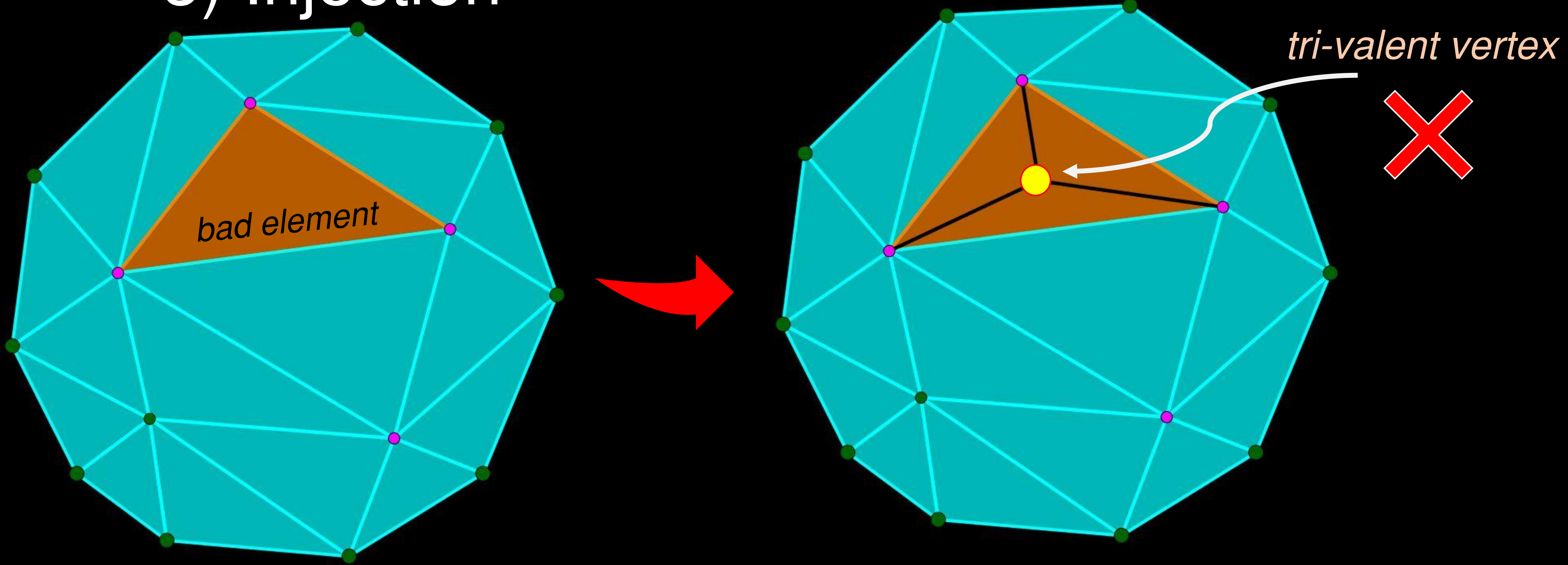
2) Ejection



The Strategy

Resampling Operators:

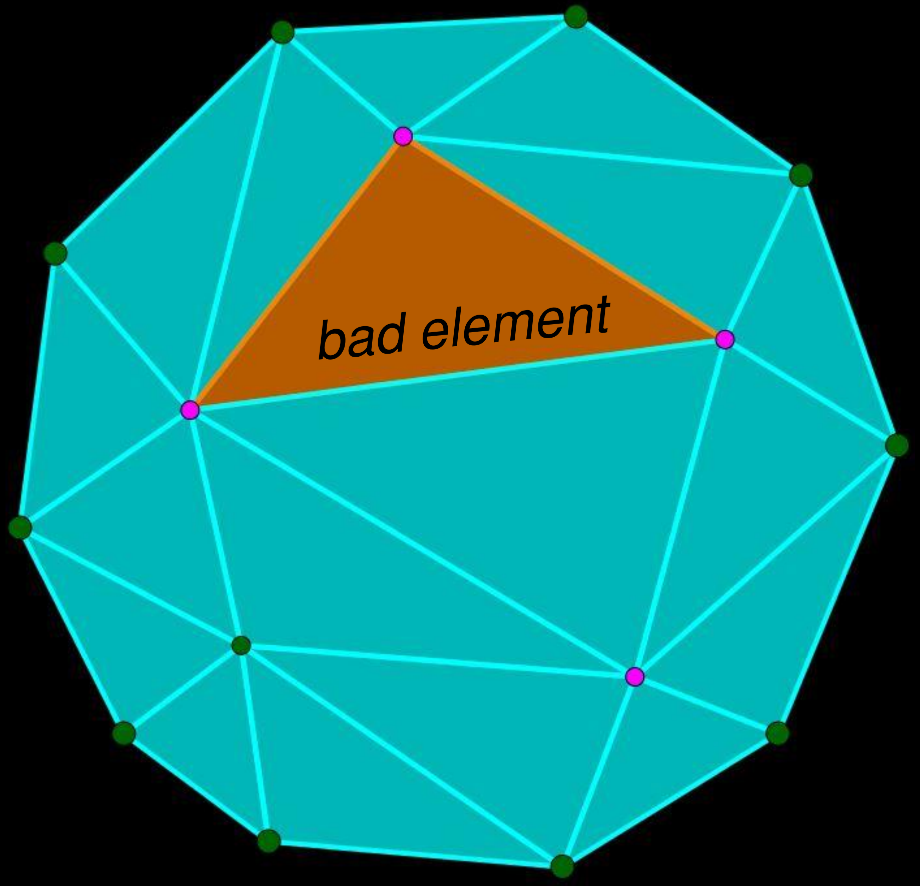
3) Injection

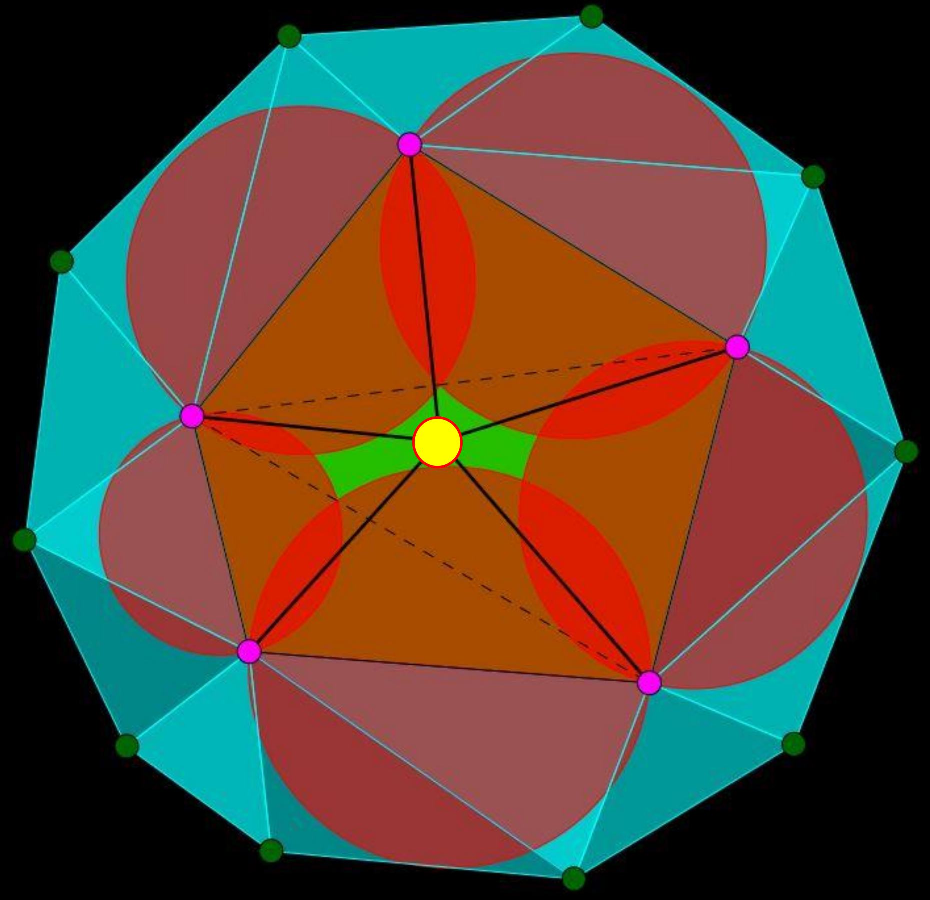


The Strategy

Resampling Operators:

3) Injection

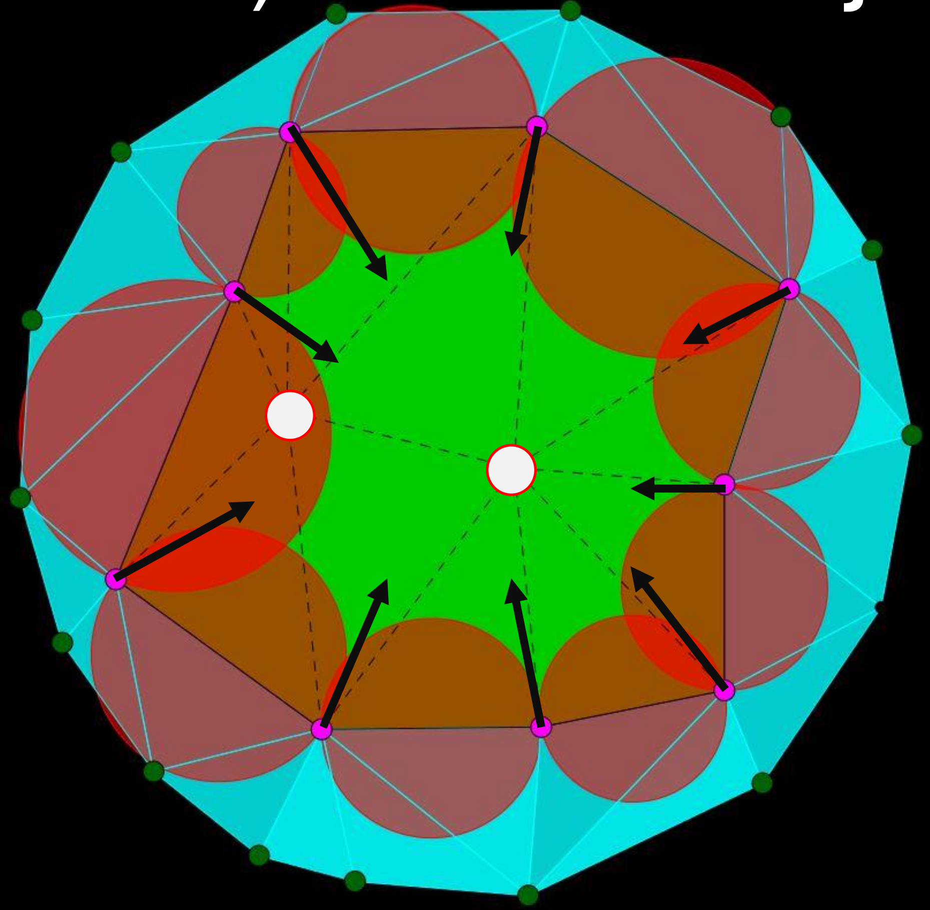


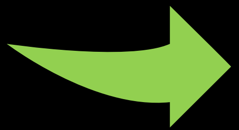


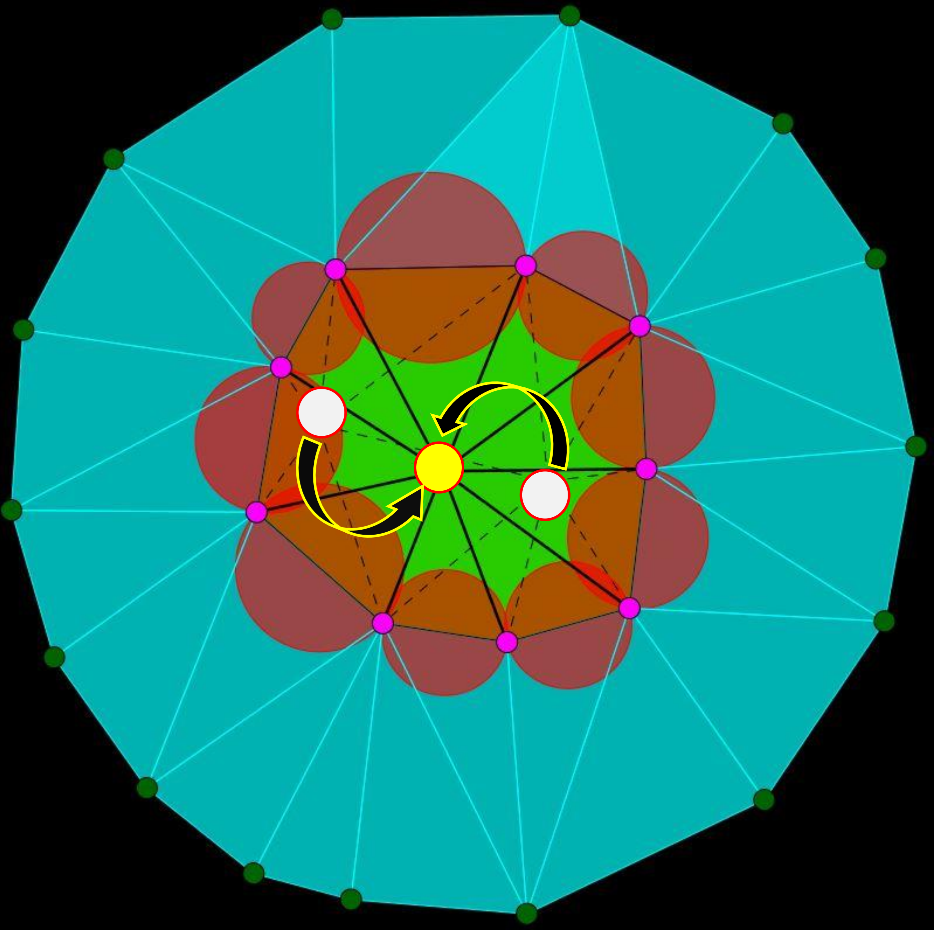
The Strategy

Resampling Operators:

4) Attractor Ejection



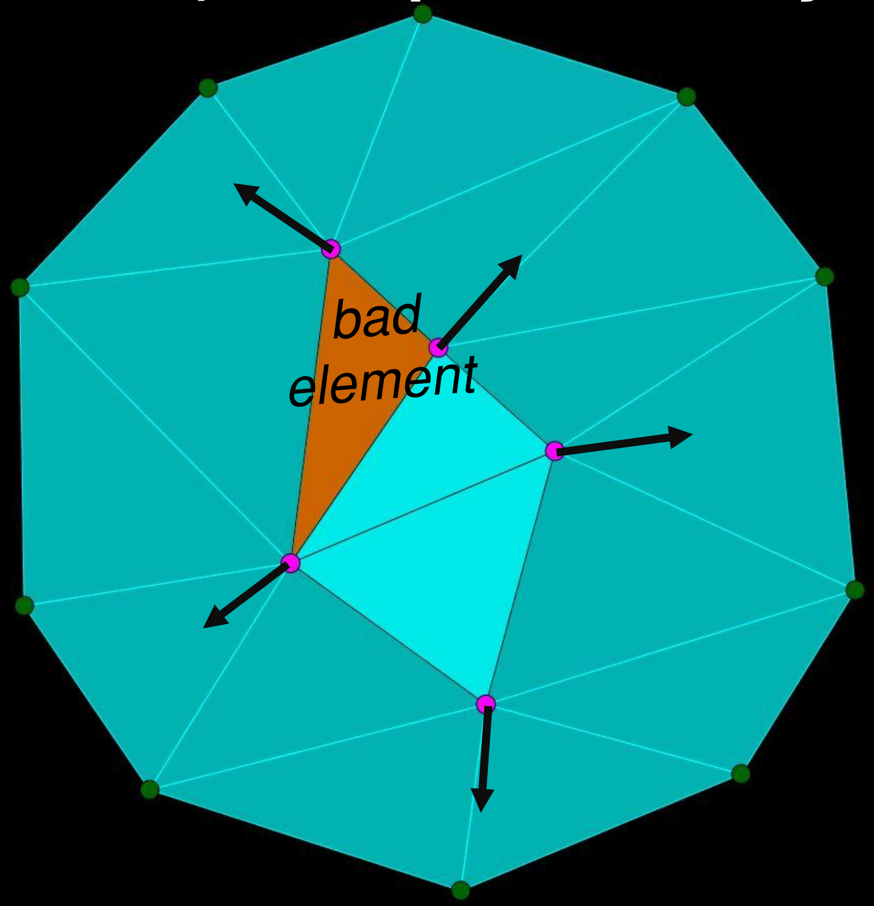


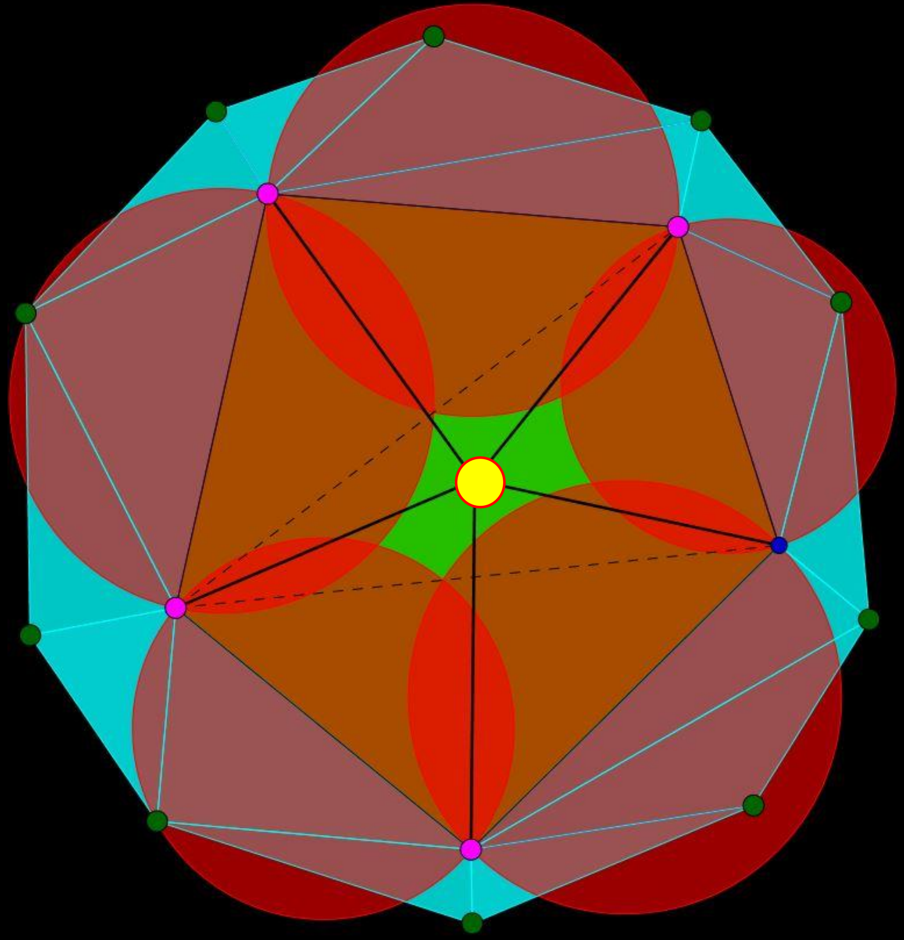


The Strategy

Resampling Operators:

5) Repeller Injection

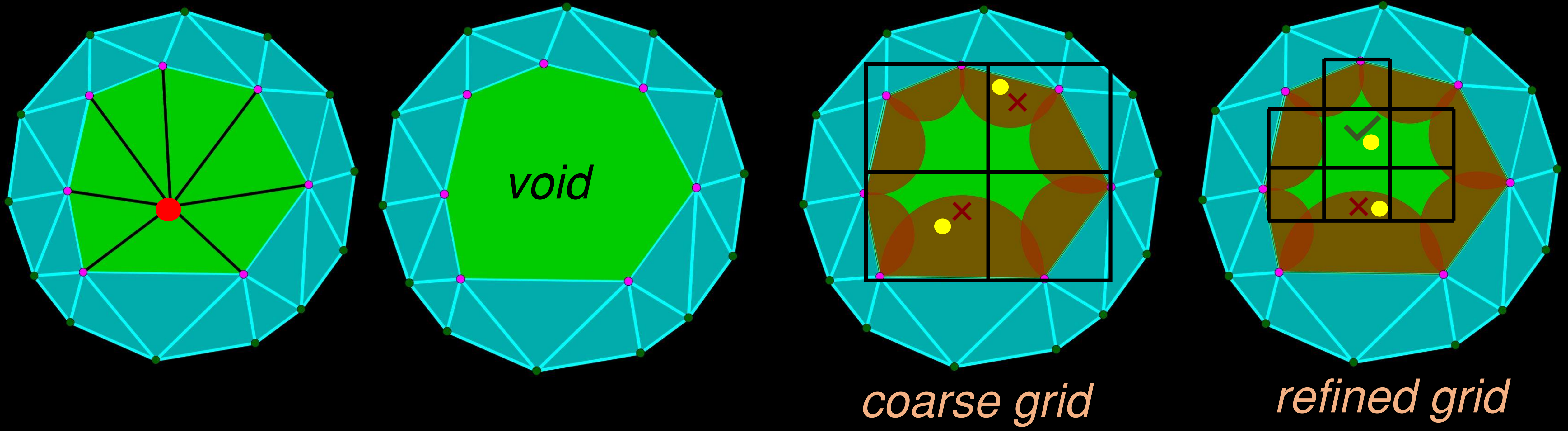




The Strategy

Sampling:

dart throwing



The Strategy

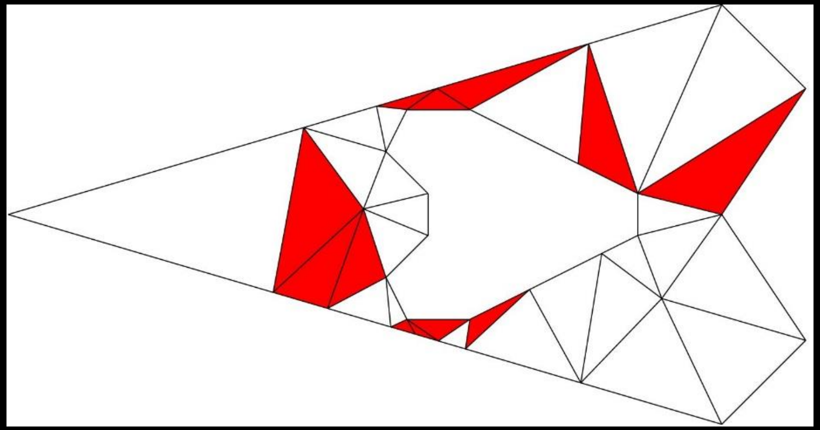
Guarantees:

- No degradation
- No repeated scenarios guarantees termination
- For curved surface, sampling from the input surface guarantees upper bound on Hausdorff distance

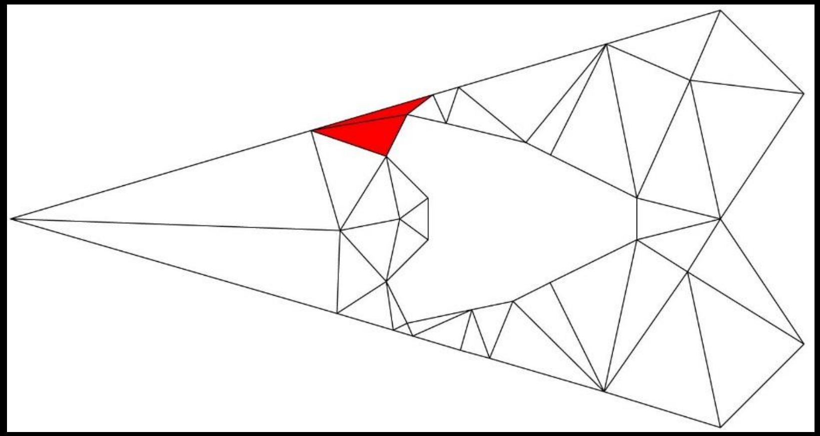
The Strategy

Limitations:

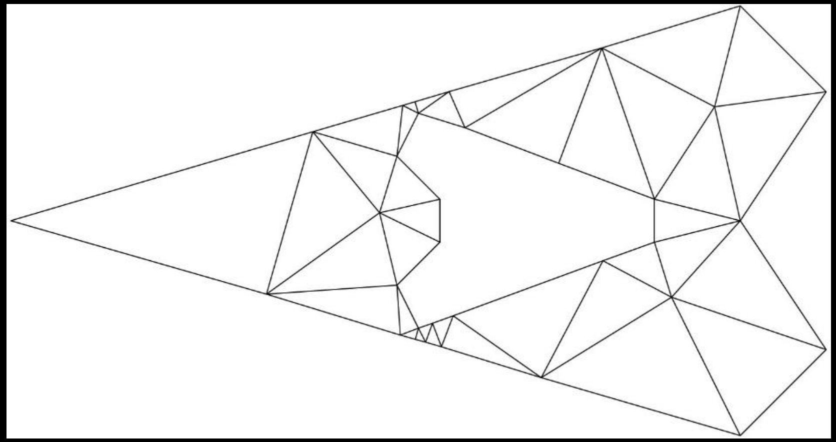
- Stuck in local minima



Input



Dead-end



Success

Summary

- Simple strategy with versatile applications
- Derived spatial representation of various qualities
- Developed a toolbox for local resampling
- Demonstrate success over wide range of applications

Thank You!

Funded:
Sandia National Labs

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